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CENTRALIZER AND NORMALIZER OF A MULTIGROUP

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Abstract: **The concept of multiset generalises the classical set, and so multigroup is an algebraic structure of multiset. In this paper we study the concept of centralizer and normalizer of a group under the context of multiset. The closure of these concept was studied under union, intersection, arithemetic addition, compposition and arithmetic multiplication among others. We have also shown that a centralizer and normalizer is not empty and are sub multi group of a multi group alongside defining the normal sub multi group of a multi group.**

Keywords: **Multiset, Multigroup, Commutative multigroup, Centralizer and Normalizer of mulrigroup.**

1. INTRODUCTION

Multisets seems to generalized the Cantorian sets ([1],[7],[8],[9],[10],[13],[14]). It was first suggested by N.G, de Bruijn ([3]) in a private communication which opposes the basic principle that an element can belong to a set only once. This study generated the study of group theory under multiset perspective which we called multi group ([5],[11],[12]).

Research is ongoing concerning multigroup as it began by a good number of researchers up untill Tella and Daniel, (2013), and was improve by Nazmul et al, (2013), Ejegwa and Ibrahim, (2017) worked on much of the properties of the classical group theory under multiset contaxt Some of the research work done is ranging from multiset operation of multi group, homomorphic nature of multi group, sub multi group and normal sub multi group, Abelian multi group, Centre of a multi group among others. ([15],[16],[17],[18])

This paper seeks to extend and to reconsider in a different perspective the study of centralizer, and normalizer of a multi group with an intension to study more results on it. That is, the operations on, the sub and normal sub multi group of these concepts and their properties of. In addition to this section we present some preliminary definitions, notations and the introductions of some of the definitions of our terms in chapter two. In chapter three we present some basic results on the concepts. We summarise our work in chapter four and then conclude.

2. PRELIMINARY DEFINITIONS AND NOTATIONS

Definition 2.1[1]. An mset A over the set X can be defined as a function $C_A: X \to \mathbb{N} = \{0,1,2,\dots\}$ where the value $C_A(x)$ denotethe number of times or multiplicity or count function of x in A. For example, Let $A = [x,x,x,y,y,y,z,z]$, then $C_A(x) = 3, C_A(y) = 3, C_A(z) = 2. C_A(x) = 0 \Rightarrow x \notin A$.

The mset M over the set X is said to be empty if $C_M(x) = 0$ for all $x \in X$. We denote the empty mset by $\emptyset$. Then $C_\emptyset(x) = 0, \forall x \in X. if$ $C_A(x) > 0, then$ $x \in A$.

Definition2.2[1]: The cardinality of a mset M denoted $|M|$ or $card(M)$ is the sum of all the multiplicities of its elements given by the expression $card(M) = \sum_{x \in X} c_A(x)$.

Definition 2.3[2]: Let M be an mset drawn from a set X. The support set of M denoted by M^* is a subset of X and $M^* = \{x \epsilon X: C_M(x) > 0\}$. that is M^* is an ordinary set. M^* is also called root set.

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Definition 2.4[1]: Equal msets. **Two msets** A and B **are said to be** equal denoted $A = B$ if and only if for any objects $x \in X, C_A(x) = C_B(x)$. This is to say that $A = B$ if the multiplicity of every element in A is equal to its multiplicities in B and conversely. Clearly, $A = B \Longrightarrow A^* = B^*$, though the converse need not hold.

For example: let $A = [a, a, b, b, c]$ and $B = [a, a, b. b, b, c, c]$ $where$ $A^* = B^* = \{a, b, c\}$ but $A \neq B$.

Definition 2.5[1] Submsets: Let X be a set and let A and B be msets over X. A is a submset of B, denoted by $A \subseteq B$ or $A \supseteq B$, if $C_A(x) \leq C_B(x)$ for all $x \in X$. Also if $A \subseteq B$ and $A \neq B$, then A is called proper submset of B denoted by $A \subset B$. In other words $A \subset B$ if $A \subseteq B$ and there exist at least one $x \in X$ such that $C_A(x) < C_B(x)$. We assert that an mset B is called the parent mset in relation to the mset A.

Definition 2.6[1]: Regular or Constant mset: A mset A is called regular or constant if all its elements are of the same multiplicities, i.e for any $x, y \in A$ such that $x \neq y, C_A(x) = C_A(y)$.

Definition 2.7[1]: The notations $\wedge$ and $\vee$:[6]. The notations $\wedge$ and $\vee$ denote the minimum and maximum operator respectively for instance $C_A(x) \wedge C_A(y) = min\{C_A(x), C_A(y)\}$ and $C_A(x) \vee C_A(y) = max\{C_A(x), C_A(y)\}$.

Definition 2.8[9]: Union ($\cup$) of msets.Let A and B be two msets over a given domain set X. The union of A and B denoted by $A \cup B$ is the mset defined by $C_{A\cup B}(x) = max\{C_A(x), C_B(x)\}$,

That is object x occurring a times in A and b times in B occur maximum $\{a, b\}$ times in $A \cup B$, if such maximum exist.

For example: Suppose $A = [b, b, c, c, c, d], B = [a, b, c, c]$, $then$ $A \cup B = [b, b, c, c, c, d]$.

Definition 2.9[9]:Intersection ($\cap$) of msets. Let A and B be two mset over a given domain set X. The intersection of two mset A and B denoted by $A \cap B$, is the mset for which $C_{A\cap B}(x) = min\{C_A(x), C_B(x)\}$for all $x \in X$.

In other words, $A \cap B$ is the smallest mset which is contained in both A and B. That is an objects x occuring a times in A and b in B, occurs minimum (a, b) times in $A \cap B$.

From the above example, $A \cap B = [b, c, c]$.

Definition 2.10[9]: Addition or sum of Mset. Let A and b be two msets over a given domain set A. The direct sum or arithmetic addition of A and B denoted by $A - B$ or $A \uplus B$ is the mset defined by $C_{A+B}(x) = C_A(x) + C_B(x)$ for all $x \in X$.

That is, an object x occurring a times in A and b times in B, occurs $a + b$ times in $A \uplus B$.

Using the above example we conclude that, $A \uplus B = [a, b, b, b, c, c, c, c, c, d]$ clearly$|A \uplus B| = |A \cup B| + |A \cap B|$.

Definition 2.11[9]: Difference of msets. Let A and B be two mset over a given domain set X. then the difference of B from A, denoted by $A - B$ is the mset such that $C_{A-B}(x) = max\{C_A(x) - C_B(x), 0\}$for all $x \in X$. If $B \subseteq A$, then $C_{A-B}(x) = C_A(x) - C_B(x)$.

It is sometimes called the arithmetic difference of B from A. If $B \nsubseteq A$ this definition still holds. It follows that the deletion of an element x from an mset A give rise to a new mset $A' = A - x$ such that $C_{A'}(x) = \{C_A(x) - 1,0\}$.

Definition 2.12[8]: Symmetric Difference. Let X be a set and A, B a mset over X. Then the symmetric difference, denoted $A\Delta B$, is defined by $C_{A\Delta B}(x) = |C_A(x) - C_A(x)|$.

Definition 2.13[8]:Compliment in msets: Let $G = \{A_1, A_2, \dots\}$ be a family of msets generated from the set X. Then, the maximum mset Z is defined by $C_Z(x) = max_{A\in G} C_A(x)$ for all $A \in G$.Complement of an mset A, denoted by $\bar{A}$, is defined by

$\bar{A} = Z - A$ and $C_{\bar{A}}(x) = \{C_Z(x) - C_A(x), for\ all\ x \in X\}$ where Z is a universal mset.

Theorem 2.14[8]. Let A and B be any mset then the following holds:

(i) $(A \cup B)^* = A^* \cup B^*$

(ii) $(A \cap B)^* = A^* \cap B^*$

Theorem 2.15 [11]. For any $M \in M(X)$, $M^* = (M^k)^* = (kM)^*$ for any $k \in N$ $such$ $that$ $k \geq 1$.

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Theorem 2.16[11]: Let $M, N \in M(X)$, $M \subseteq N \rightarrow M^* \subseteq N^*$

Definition 2.17[6]: Let X be a group. An mset A over X is said to be a multigroup (mgroup for short) over X if **the** count function $C_A(x)$ satisfied the following conditions:

(i) $C_A(xy) \geq C_A(x) \wedge C_A(y) \forall\, x, y \in X$.

(ii) $C_A(x^{-1}) \geq C_A(x) \forall x \in X$

It follows immediately that:

$$C_A(x^{-1}) = C_A(x)\ \forall\ x \in X$$

We denote the set of all mgroups over X by $MG(X)$.

Theorem 2.18[5]: Let X be a group. **If** G **is a mgroup over** $\boldsymbol{X}$**, then**

(i) $C_G(e) \geq C_G(x), \forall\ x \in X$ and e identity in X.

(ii) $C_G(x^n) \geq C_G(x), \forall\ x, \forall\ n \in \mathbb{N}$.

(iii) $C_G(x^{-1}) = C_G(x), \forall\ x \in X$

(iv) $G = G^{-1}$

Definition 2.19[5]; Composition and Inverse. Let $A, B \in MG(X)$, then we call

(i) $A \circ B$ as the composition between two mgroups defined as

$C_{A \circ B}(x) = \vee\{C_A(y) \wedge C_B(z): y, z \in X \ni yz = x\}$ and

(ii) A^{-1} is called the inverse of mgroup A and defined as $C_{A^{-1}}(x) = C_A(x^{-1})$ **for all** $\boldsymbol{x} \in \boldsymbol{X}$**.**

Theorem 2.20[5]. Let $A, B \in MG(X)$, then the following assertions holds:

(i) $[A^{-1}]^{-1} = A$

(ii) $A \subseteq B \Longrightarrow A^{-1} \subseteq B^{-1}$

(iii) $(A \circ B)^{-1} = B^{-1} \circ A^{-1}$

(iv) $[\cap_{i \in I} A_i]^{-1} = \cap_{i \in I} [A_i^{-1}]$

(v) $[\cup_{i \in I} A_i]^{-1} = \cup_{i \in I} [A_i^{-1}]$

(vi) $(A \circ B) \circ C = A \circ (B \circ C)$

Theorem 2.21[5]. Let $A, B \in MG(X)$. Then $A \cap B \in MG(X)$.

Theorem 2.21[5]. Let $A, B \in MG(X)$. Then $A \cup B \notin MG(X)$.

Theorem 2.22[17]: If $A, B \in MG(X)$, then the sum of A and B is a multigroup of X.

Theorem 2.23[17]: Let $A \in MG(X)$ and if $x, y \in X$ with $C_A(x) \neq C_A(y)$, then

$$C_A(xy) = C_A(yx) = C_A(x) \wedge C_A(y)$$

Theorem 2.24[17]: Let $A \in MG(X)$ and B be a nonempty submultiset of A. Then the following statements are equivalent.

(i) B is a submultigroup of A.

(ii) $C_B(yx) = C_{\boldsymbol{B}}(x) \wedge C_B(y)$ and $C_B(x^{-1}) = C_B(x) \forall\ x, y \in X$.

(iii) $C_B(xy^{-1}) = C_{\boldsymbol{B}}(x) \wedge C_B(y) \forall\ x, y \in X$.

Definition 2.25[5]: Let $A \in MG(X)$ and if $\forall\ x, y \in X$, we defined a commutative multigroup or an Abelian multigroup as $C_A(xy) = C_A(yx)$.

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For example; If $X = \{e, a, b, c\}$ be a commutative group then let $A = \{e, a, b, c\}_{4,3,3,2}$. Hence A is an Abelian or commutative multigroup.

Theorem 2.26[5]: Let B be a commutative multigroup of a group X. Then,

(i) $C_B([x, y]) = C_B(e)$.

(ii) $C_B([x, y]) = C_B(x)$.

3. CENTRALIZER AND NORMALIZER OF A MULTIGROUP

Definition 3.1: Let M be a multigroup over a group X. Then the centralizer of M, denoted $C(M)$ and defined

$C(M) = \{x \in X / C_M(xy) = C_M(xy)\ \forall y \in X\}$.

Example: Suppose the group $X = \{0,1,2\} = Z_3$ and let $M \in MG(X)$, that is $M = \{0,1,2\}_{3,2,1}$. Clearly $C(M)$ is can be shown.

Thus $C(M)$ is a the centre of the multigroup M.

We denote $\mathcal{C}(M)$ as the class of all centralizers of a multigroup.

Proposition 3.2: Let $M \in MG(X)$. Then the centre of M, $C(M)$ is a subgroup of X.

Proof: Let $C_M(x) \neq \emptyset$. Then at least $e \in C(M)$. Now let $x, y \in C(M)$.

Since $C_M([x, y]) = C_M(e)$ and for all $x, y \in X$.

Consequently, $C_M([x, y]) = C_M(e) = C_A(xyx^{-1}y^{-1})$

$$\geq C_M(xy) \wedge C_M(x^{-1}y^{-1})$$
$$= C_M(xx^{-1}) \wedge C_M(yy^{-1})$$
$$= C_M(e) \wedge C_M(e)$$
$$= C_M(e)$$

Again, let $x, y \in C(M)$ we want to show that $xy \in C(M)$ and if $x \in C(M)$ then $x^{-1} \in C(M)$.

Now

$$C_M([x, y]) = C_A(xyx^{-1}y^{-1})$$
$$\geq C_M(xy) \wedge C_M(x^{-1}y^{-1})$$
$$= C_M(xy) \wedge C_M(xy)^{-1}$$
$$= C_M(xy) \wedge C_M(xy)$$
$$= C_M(xy)$$

This implies that $xy \in C(M)$ and

$$C_M([x, x^{-1}]) = C_A(xxx^{-1}x^{-1})$$
$$\geq C_M(xx) \wedge C_M(x^{-1}x^{-1})$$
$$= C_M(xx^{-1})^{-1} \wedge C_M(xx^{-1})^{-1}$$
$$= C_M(x^{-1}x) \wedge C_M(x^{-1}x)$$
$$= C_M(e) \wedge C_M(e)$$
$$= C_M(e)$$

Showing $x^{-1} \in C(M)$. Hence $C(M)$ is a subgroup of X.

Proposition 3.3: Let $M, N \in MG(X)$. If $M \cap N \in MG(X)$ then $C(M) \cap C(N) \in \mathcal{C}(M)$.

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Proof: Let $M \in MG(X)$. Then $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\ x, y \in M$.

$N \in MG(X)$. Then $C_N(xy) \geq C_N(x) \wedge C_N(y)$ and $C_N(x)^{-1} = C_N(x) \forall\ x, y \in N$.

Now, $C_{M\cap N}(xy) = \wedge\{C_{M\cap N}(x), C_{M\cap N}(y)\}$

$$\geq \wedge\{[C_M(x) \wedge C_M(y)], [C_N(x) \wedge C_N(y)]\}$$

$$\geq C_M(x) \wedge C_M(y) \wedge C_N(x) \wedge C_N(y)$$

$$= [C_M(x) \wedge C_N(y)] \wedge [C_M(x) \wedge C_N(y)]$$

$$= C_{M\cap N}(x) \wedge C_{M\cap N}(y)$$

And $C_{M\cap N}(x)^{-1} = C_{M\cap N}(x)$

Therefore $M \cap N \in MG(X)$.

For $C(M) \cap C(N) \in \mathcal{C}(M)$,

Then $C_{C(M)\cap C(N)}(xy) = \{C_{C(M)\cap C(N)}(xy) \wedge C_{C(M)\cap C(N)}(xy)\} \forall\ x, y \in M$.

$$\geq \wedge\{[C_{C(M)}(x) \wedge C_{C(N)}(y)], [C_{C(M)}(x) \wedge C_{C(N)}(y)]\}$$

$$= C_{C(M)}(x) \wedge C_{C(M)}(y) \wedge C_{C(M)}(x) \wedge C_{C(N)}(y)$$

$$= [C_{C(M)}(x) \wedge C_{C(N)}(x)] \wedge [C_{C(M)}(y) \wedge C_{C(N)}(y)]$$

$$= [C_{C(M)}(y) \wedge C_{C(N)}(y)] \wedge [C_{C(M)}(x) \wedge C_{C(N)}(x)]$$

$$= C_{C(M)\cap C(N)}(y) \wedge C_{C(M)\cap C(N)}(x)$$

$$= C_{C(M)\cap C(N)}(yx)$$

Therefore $C(M) \cap C(N) \in \mathcal{C}(M)$.

Proposition 3.4: Let $M \in MG(X)$ then $C(M) \in MG(X)$.

Proof: Since $M \in MG(X)$, then M^* is a group and $\forall\ x, y \in X$.

Now $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\ x, y \in M$. Also $C(M) \in MG(X)$ shows that $C(M)^*$ is the centralizer of the group M then

$C_{C(M)}(xy) \geq C_{C(M)}(x) \wedge C_{C(M)}(y)$ and $C_{C(M)}(x)^{-1} = C_{C(M)}(x)\ \forall\ x, y \in X$.

Hence $C(M) \in MG(X)$.

Proposition 3.5: Let $M, N \in MG(X)$. If $M \cup N \in MG(X)$ then $C(M) \cup C(N) \in \mathcal{C}(M)$.

Proof: Let $M \in MG(X)$. Then $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\ x, y \in M$.

$N \in MG(X)$. Then $C_N(xy) \geq C_N(x) \wedge C_N(y)$ and $C_N(x)^{-1} = C_N(x) \forall\ x, y \in N$.

Now, $C_{M\cup N}(xy) = \vee\{C_{M\cup N}(x), C_{M\cup N}(y)\}$

$$\geq \vee\{[C_M(x) \wedge C_M(y)], [C_N(x) \wedge C_N(y)]\}$$

$$\geq C_M(x) \wedge C_M(y) \vee C_N(x) \wedge C_N(y)$$

$$= [C_M(x) \wedge C_N(y)] \vee [C_M(x) \wedge C_N(y)]$$

$$\neq C_{M\cup N}(x) \vee C_{M\cup N}(y)$$

Therefore $M \cup N \notin MG(X)$.

For $C(M) \cup C(N) \notin \mathcal{C}(M)$,

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then $C_{C(M)\cup C(N)}(xy) = \{C_{C(M)\cup C(N)}(xy) \wedge C_{C(M)\cup C(N)}(xy)\}\ \forall\, x, y \in M.$

$$\geq \vee\{[C_{C(M)}(x) \wedge C_{C(N)}(y)], [C_{C(M)}(x) \wedge C_{C(N)}(y)]\}$$

$$= C_{C(M)}(x) \wedge C_{C(M)}(y) \vee C_{C(M)}(x) \wedge C_{C(N)}(y)$$

$$= [C_{C(M)}(x) \wedge C_{C(N)}(x)] \vee [C_{C(M)}(y) \wedge C_{C(N)}(y)]$$

$$= [C_{C(M)}(y) \wedge C_{C(N)}(y)] \vee [C_{C(M)}(x) \wedge C_{C(N)}(x)]$$

$$= C_{C(M)\cup C(N)}(y) \vee C_{C(M)\cup C(N)}(x)$$

$$\neq C_{C(M)\cup C(N)}(yx)$$

Therefore $C(M) \cap C(N) \notin \mathcal{C}(M)$.

Remark: If M^* is an abelian group and $M \in MG(X)$. Then $C(M) = M$.

Proposition 3.6: Let $M, N \in MG(X)$. If $M + N \in MG(X)$ then $C(M) + C(N) \in \mathcal{C}(M)$.

Proof: Let $M \in MG(X)$. Then $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\, x, y \in M$.

$N \in MG(X)$. Then $C_N(xy) \geq C_N(x) \wedge C_N(y)$ and $C_N(x)^{-1} = C_N(x) \forall\, x, y \in N$.

Now, $C_{M+N}(xy) = \{C_M(xy) + C_N(xy)\}$

$$\geq \{[C_M(x) \wedge C_M(y)] + [C_N(x) \wedge C_N(y)]\}$$

$$= C_M(x) \wedge C_N(x) + C_M(y) \wedge C_N(y)$$

$$= [C_M(x) \wedge C_N(x)] + [C_M(y) \wedge C_N(y)]$$

$$= C_{M+N}(x) \wedge C_{M+N}(y)$$

And $C_{M+N}(x)^{-1} = C_{M+N}(x)$

Therefore $M + N \in MG(X)$.

For $C(M) + C(N) \in \mathcal{C}(M)$,

then $C_{C(M)+C(N)}(xy) = \{C_{C(M)+C(N)}(x) \wedge C_{C(M)+C(N)}(y)\} \forall\, x, y \in M.$

$$\geq \{[C_{C(M)}(x) + C_{C(N)}(x)] \wedge [C_{C(M)}(y) + C_{C(N)}(y)]\}$$

$$= C_{C(M)}(x) \wedge C_{C(M)}(y) + C_{C(M)}(x) \wedge C_{C(N)}(y)$$

$$= [C_{C(M)}(x) \wedge C_{C(N)}(x)] + [C_{C(M)}(y) \wedge C_{C(N)}(y)]$$

$$= [C_{C(M)}(y) \wedge C_{C(N)}(y)] + [C_{C(M)}(x) \wedge C_{C(N)}(x)]$$

$$= C_{C(M)+C(N)}(y) \wedge C_{C(M)+C(N)}(x)$$

$$= C_{C(M)+C(N)}(yx)$$

Therefore $C(M) + C(N) \in \mathcal{C}(M)$.

Proposition 3.7: Let $M, N \in MG(X)$. If $M.N \in MG(X)$ then $C(M).C(N) \in \mathcal{C}(M)$.

Proof: Let $M \in MG(X)$. Then $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\, x, y \in M$.

$N \in MG(X)$. Then $C_N(xy) \geq C_N(x) \wedge C_N(y)$ and $C_N(x)^{-1} = C_N(x) \forall\, x, y \in N$.

Now, $C_{M.N}(xy) = \{C_M(xy).C_N(xy)\}$

$$\geq \{[C_M(x) \wedge C_M(y)].[C_N(x) \wedge C_N(y)]\}$$

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$= C_M(x) \wedge C_N(x). C_M(y) \wedge C_N(y)$

$= [C_M(x) \wedge C_N(x)].[C_M(y) \wedge C_N(y)]$

$= C_{M.N}(x) \wedge C_{M.N}(y)$

And $C_{M.N}(x)^{-1} = C_{M.N}(x)$

Therefore $M.N \in MG(X)$.

For $C(M).C(N) \in \mathcal{C}(M)$,

then $C_{C(M).C(N)}(xy) = \{C_{C(M).C(N)}(x) \wedge C_{C(M).C(N)}(y)\} \forall\, x, y \in M$.

$\geq \{[C_{C(M)}(x). C_{C(N)}(x)] \wedge [C_{C(M)}(y). C_{C(N)}(y)]\}$

$= C_{C(M)}(x) \wedge C_{C(M)}(y). C_{C(M)}(x) \wedge C_{C(N)}(y)$

$= [C_{C(M)}(x) \wedge C_{C(N)}(x)].[C_{C(M)}(y) \wedge C_{C(N)}(y)]$

$= [C_{C(M)}(y) \wedge C_{C(N)}(y)].[C_{C(M)}(x) \wedge C_{C(N)}(x)]$

$= C_{C(M).C(N)}(y) \wedge C_{C(M).C(N)}(x)$

$= C_{C(M).C(N)}(yx)$

Therefore $C(M).C(N) \in \mathcal{C}(M)$.

Proposition 3.8: Let $M, N \in MG(X)$. If $MoN \in MG(X)$ then $C(M)oC(N) \in \mathcal{C}(M)$.

Proof: Let $M \in MG(X)$. Then $C_M(xy) \geq C_M(x) \wedge C_M(y)$ and $C_M(x)^{-1} = C_M(x)\ \forall\, x, y \in M$.

$N \in MG(X)$. Then $C_N(xy) \geq C_N(x) \wedge C_N(y)$ and $C_N(x)^{-1} = C_N(x) \forall\, x, y \in N$.

Now, $C_{MoN}(xy) = \{C_M(xy) o C_N(xy)\}$

$\geq \{[C_M(x) \wedge C_M(y)] o [C_N(x) \wedge C_N(y)]\}$

$= C_M(x) \wedge C_N(x) o C_M(y) \wedge C_N(y)$

$= [C_M(x) \wedge C_N(x)] o [C_M(y) \wedge C_N(y)]$

$= C_{MoN}(x) \wedge C_{MoN}(y)$

And $C_{MoN}(x)^{-1} = C_{MoN}(x)$

Therefore $MoN \in MG(X)$.

For $C(M)oC(N) \in \mathcal{C}(M)$,

then $C_{C(M)oC(N)}(xy) = \{C_{C(M)oC(N)}(x) \wedge C_{C(M)oC(N)}(y)\} \forall\, x, y \in M$.

$\geq \{[C_{C(M)}(x) o C_{C(N)}(x)] \wedge [C_{C(M)}(y) o C_{C(N)}(y)]\}$

$= C_{C(M)}(x) \wedge C_{C(M)}(y) o C_{C(M)}(x) \wedge C_{C(N)}(y)$

$= [C_{C(M)}(x) \wedge C_{C(N)}(x)] o [C_{C(M)}(y) \wedge C_{C(N)}(y)]$

$= [C_{C(M)}(y) \wedge C_{C(N)}(y)] o [C_{C(M)}(x) \wedge C_{C(N)}(x)]$

$= C_{C(M)oC(N)}(y) \wedge C_{C(M)oC(N)}(x)$

$= C_{C(M)oC(N)}(yx)$

Therefore $C(M)oC(N) \in \mathcal{C}(M)$.

Proposition 3.9: Let $M, N \in MG(X)$, then $C(M)oC(N) \subseteq C(MoN)$.

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Proof: Let $m \in C(M)$ and $n \in C(N)$, for all $x \in X$.

Now $C_{MoN}((mn)x) = \vee\{C_M(y) \wedge C_N(z)\ \forall y, z \in X, mnx = yz\}$

$$= \vee\{C_M(mnxz^{-1}) \wedge C_N(z)\ \forall z \in X, mnx = yz\}$$

$$= \vee\{C_M(nxmz^{-1}) \wedge C_N(z)\ \forall z \in X, mnx = yz\}$$

$$= \vee\{C_M(y) \wedge C_N(z)\ \forall y, z \in X, nxm = yz\}$$

$$= \vee\{C_M(y) \wedge C_N(y^{-1}nxm)\ \forall y \in X, nxm = yz\}$$

$$= \vee\{C_M(y) \wedge C_N(y^{-1}xnm)\ \forall y \in X, nxm = yz\}$$

$$= \vee\{C_M(y) \wedge C_N(z)\ \forall y, z \in X, xmn = yz\}$$

$$= C_{MoN}(x(mn))$$

Similarly, $C_{MoN}(x(mn)) = C_{MoN}((mn)x)$.

Hence $mn \in C(MoN)$.

Thus $C(M)oC(N) \subseteq C(MoN)$.

Proposition 3.10: Let $M \in MG(X)$, then $C(M) \subseteq X$.

Proof: It is obvious that $C(M) \neq \emptyset$ and since at least $e \in C(M)$. Let $m, n \in C(M)$. We want to show that $mn \in C(M)$.

Then $C_M(mn) = C_M(nm) \geq C_M(m) \wedge C_M(n) = C_M(n) \wedge C_M(m)$

$$\Leftrightarrow C_M(mm^{-1}nn^{-1}) = C_M(e)$$

$$\Leftrightarrow C_M([m, n]) = C_M(e)$$

Thus $mn \in C(M)$.

Also, let $m \in C(M)$. Then $C_M([m, n]) = C_M(e)\ \forall\, n \in X$. Hence

$$C_M([m^{-1}, n]) = C_M(mn^{-1}m^{-1}n) = C_M(mn^{-1}m^{-1}nmm^{-1})$$

$$= C_M(n^{-1}m^{-1}nmm^{-1}m) = C_M([n, m])$$

$$= C_M([m, n]^{-1}) = C_M([m, n]) = C_M(e)$$

Thus $m^{-1} \in C(M)$.

Therefore $C(M) \subseteq X$.

Remark: Let $M \in MG(X)$, then $C(M) = A^*$. If A is commutative or regular multigroup. Otherwise $C(M) \subseteq A$.

Proposition 3.11: Let $A \in AMG(X)$ be abelian, then

(i) $C_A([x, y]) = C_A(e)$

(ii) $C_A([x, y]) \geq C_A(x)$

Where (e) is the identity element of X and $[x, y]$ is the commutator of x *and* y *in* X.

Proof: (i) Let $x, y \in X$ such that x *and* y commutes with each other. Now

$$C_A([x, y]) = C_A(x^{-1}y^{-1}xy) = C_A(x^{-1}xy^{-1}y)$$

$$\geq C_A(x^{-1}x) \wedge C_A(y^{-1}y)$$

$$= C_A(e) \wedge C_A(e)$$

$$= C_A(e)$$

$\rightarrow C_A([x, y]) \geq C_A(e)$ and

$$C_A(e) = C_A(xyx^{-1}y^{-1}) \geq C_A\big((xyx^{-1}y^{-1})e\big)$$

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$$= C_A\big((xyx^{-1}y^{-1})(xyx^{-1}y^{-1})\big)$$

$$\geq C_A(xyx^{-1}y^{-1}) \wedge C_A(xyx^{-1}y^{-1})$$

$$= C_A(x^{-1}y^{-1}xy)$$

$\Rightarrow C_A(e) \geq C_A([x,y])$.

Hence $C_A([x,y]) = C_A(e)$.

(i) $C_A([x,y]) = C_A(x^{-1}y^{-1}xy) = C_A(x^{-1}y^{-1}xy) \geq C_A(x^{-1}) \wedge C_A(y^{-1}xy)$

$$\geq C_A(x) \wedge C_A(x) = C_A(x)$$

Thus $C_A([x,y]) \geq C_A(x)$

Proposition 3.12: Let $A \in MG(X)$ be abelian and $n \in \boldsymbol{N}$. Then $C_A((xy)^n) = C_A(x^n y^n)$ for all $x, y \in X$.

Proof: Let $x, y \in X$. We have $C_A((xy)^n) = C_A(xy \ldots xyxyxy) = C_A(xy \ldots xyxy^2x[x,y])$

$$\geq C_A(xy \ldots xyxy^2x) \wedge C_A([x,y]) = C_A(x^2y \ldots xyxy^2) = C_A(x^2y \ldots xy^3x) = C_A(x^2y \ldots xy^3x[x,y])$$

$\geq C_A(x^2y \ldots xy^3x) \geq \cdots \geq C_A(x^{n-1}yxy^{n-1}) = C_A(x^{n-1}xy^n[x,y^{n-1}]) \geq C_A(x^{n-1}y^nx) = C_A(x^ny^n)$.

$\Rightarrow C_A((xy)^n) = C_A(x^ny^n)$.

Also,

$C_A(x^ny^n) = C_A(x^{n-1}y^nx) = C_A(x^{n-1}yxy^n[y^{n-1},x]) \geq C_A(x^{n-1}yxy^{n-1}) \geq \cdots \geq C_A(xy \ldots xyxy^2x) =$
$C_A(xy \ldots xyxyxy[x,y]) \geq C_A(xy \ldots xyxyxy[x,y]) = C_A((xy)^n)$.

$\Rightarrow C_A(x^ny^n) = C_A((xy)^n)$.

Hence $C_A((xy)^n) = C_A(x^ny^n)$.

Definition 3.13: Let $A, B \in MG(X)$, such that $A \subseteq B$, then we defined the Normalizer of A in B as

$$N(A) = \{x \in X /\ C_A(x^{-1}yx) = C_A(y)\ \forall\ y \in X\}$$

Example: Let $X = \{e,a,b,c\}$ be a group under the multiplicative operation such that $ab = c, bc = a, ac = b$ and $a^2 = b^2 = c^2 = e$. Also let $B = \{e,a,b,c\}_{4,3,2,2}$ be a multigroup and $A = \{e,a,b,c\}_{3,2,1,1}$ be a submultigroup B. Then the normalizer of $A, N(A)$ is given by;

$C_A(a^{-1}ba) = C_A(aba) = C_A\big((ac)c\big) = C_A(ca) = C_A(b) = 1$ by definition.

Hence $N(A)$ is a normalizer of A in B.

We denote the class of all normalizers of A in B as $\mathrm{N}(A)$.

Proposition 3.14: Let M be a multigroup and if M_1 and $M_2$are normal sub multi group of M. Then $N(M_1) \cap N(M_2) \in \mathrm{N}(A)$.

Proof: Since $N(M_1), N(M_2) \in \mathrm{N}(A)$, we have

$$C_{N(M_1)}(x^{-1}yx) = C_{N(M_1)}(y)\ \forall\ x, y \in M^*$$

$$C_{N(M_2)}(x^{-1}yx) = C_{N(M_2)}(y)\ \forall\ x, y \in M^*$$

Then $C_{N(M_1)\cap N(M_2)}(x^{-1}yx) = \wedge\{C_{N(M_1)}(x^{-1}yx), C_{N(M_2)}(x^{-1}yx)\}$

$$= \wedge\{C_{N(M_1)}(y), C_{N(M_2)}(y)\}$$

$$= C_{N(M_1)}(y) \wedge C_{N(M_2)}(y)\ \forall\ x \in {M_1}^*, y \in {M_1}^* \cap {M_2}^*$$

$$= C_{N(M_1)\cap N(M_2)}(y)\ \ \forall\ x \in {M_1}^*, y \in {M_1}^* \cap {M_2}^*$$

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Hence $N(M_1) \cap N(M_2) \in \text{Ɲ}(A)$.

Proposition 3.15: Let M be a multigroup and if M_1 and $M_2$are normal submultigroup of M. Then $N(M_1) \cup N(M_2) \notin \text{Ɲ}(A)$.

Proof: Since $N(M_1), N(M_2) \in \text{Ɲ}(A)$, we have

$$C_{N(M_1)}(x^{-1}yx) = C_{N(M_1)}(y) \ \forall\, x, y \in M^*$$

$$C_{N(M_2)}(x^{-1}yx) = C_{N(M_2)}(y) \ \forall\, x, y \in M^*$$

Then $C_{N(M_1)\cup N(M_2)}(x^{-1}yx) = \vee\{C_{N(M_1)}(x^{-1}yx), C_{N(M_2)}(x^{-1}yx)\}$

$$= \vee\{C_{N(M_1)}(y), C_{N(M_2)}(y)\}$$

$$\neq C_{N(M_1)}(y) \vee C_{N(M_2)}(y) \ \forall\, x \in {M_1}^*, y \in {M_1}^* \cup {M_2}^*$$

$$\neq C_{N(M_1)\cup N(M_2)}(y) \ \ \forall\, x \in {M_1}^*, y \in {M_1}^* \cup {M_2}^*$$

Hence $N(M_1) \cup N(M_2) \notin \text{Ɲ}(A)$.

Proposition 3.16: Let M be a multigroup and if M_1 and M_2 are normal submultigroup of M. Then $N(M_1) + N(M_2) \in \text{Ɲ}(A)$.

Proof: Since $N(M_1), N(M_2) \in \text{Ɲ}(A)$, we have

$$C_{N(M_1)}(x^{-1}yx) = C_{N(M_1)}(y) \ \forall\, x, y \in M^*$$

$$C_{N(M_2)}(x^{-1}yx) = C_{N(M_2)}(y) \ \forall\, x, y \in M^*$$

Then $C_{N(M_1)+N(M_2)}(x^{-1}yx) = \{C_{N(M_1)}(x^{-1}yx) + C_{N(M_2)}(x^{-1}yx)\}$

$$= \{C_{N(M_1)}(y) + C_{N(M_2)}(y)\}$$

$$= C_{N(M_1)}(y) + C_{N(M_2)}(y) \ \forall\, x \in {M_1}^*, y \in {M_1}^* + {M_2}^*$$

$$= C_{N(M_1)+N(M_2)}(y) \ \ \forall\, x \in {M_1}^*, y \in {M_1}^* + {M_2}^*$$

Hence $N(M_1) + N(M_2) \in \text{Ɲ}(A)$.

Proposition 3.17: Let M be a multigroup and if M_1 and $M_2$are normal submultigroup of M. Then $N(M_1).N(M_2) \in \text{Ɲ}(A)$.

Proof: Since $N(M_1), N(M_2) \in \text{Ɲ}(A)$, we have

$$C_{N(M_1)}(x^{-1}yx) = C_{N(M_1)}(y) \ \forall\, x, y \in M^*$$

$$C_{N(M_2)}(x^{-1}yx) = C_{N(M_2)}(y) \ \forall\, x, y \in M^*$$

Then $C_{N(M_1).N(M_2)}(x^{-1}yx) = \{C_{N(M_1)}(x^{-1}yx).C_{N(M_2)}(x^{-1}yx)\}$

$$= \{C_{N(M_1)}(y).C_{N(M_2)}(y)\}$$

$$= C_{N(M_1)}(y).C_{N(M_2)}(y) \ \forall\, x \in {M_1}^*, y \in {M_1}^*.{M_2}^*$$

$$= C_{N(M_1).N(M_2)}(y) \ \ \forall\, x \in {M_1}^*, y \in {M_1}^*.{M_2}^*$$

Hence $N(M_1).N(M_2) \in \text{Ɲ}(A)$.

Proposition 3.18: Let M be a multigroup and if M_1 and M_2 are normal sub multigroup of M. Then $N(M_1)oN(M_2) \in \text{Ɲ}(A)$.

Proof: Since $N(M_1), N(M_2) \in \text{Ɲ}(A)$, we have

$$C_{N(M_1)}(x^{-1}yx) = C_{N(M_1)}(y) \ \forall\, x, y \in M^*$$

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$$C_{N(M_2)}(x^{-1}yx) = C_{N(M_2)}(y)\ \forall\ x, y \in M^*$$

Then,

$$C_{N(M_1)oN(M_2)}(x^{-1}yx) = \vee\{C_{N(M_1)}(x^{-1}yx) \wedge C_{N(M_2)}(x^{-1}yx)\ \forall\ x \in {M_1}^*, y \in {M_1}^* o {M_2}^*, x^{-1}yx = y\}$$

$$= \wedge\{C_{N(M_1)}(y) \wedge C_{N(M_2)}(y)\ \forall\ x \in {M_1}^*, y \in {M_1}^* o {M_2}^*, x^{-1}yx = y\}$$

$$= C_{N(M_1)oN(M_2)}(y)\ \ \forall\ x \in {M_1}^*, y \in {M_1}^* o {M_2}^*$$

Hence $N(M_1)oN(M_2) \in \mathrm{N}(A)$.

Proposition 3.19: Let $A \in MG(X)$, and $N(A)$ a normalizer of A over X. Then the normalizer of A is a subgroup of X.

Proof: $N(A) \neq \emptyset$. Then at least $e \in N(A)$. Let $x, y \in N(A)$. Now $C_A(x^{-1}yx) = C_A(y)$ and $C_A(z^{-1}yz) = C_A(y)\ \forall y \in X$.

Consequently, $C_A\big((xz)^{-1}y(xz)\big) = C_A(z^{-1}x^{-1}yxz)$

$$= C_A(x^{-1}yx) \wedge C_A(z^{-1}yz)$$

$$= C_A(y) \wedge C_A(y)$$

$$= C_A(y)$$

Then $xz \in N(A)$.

Again $C_A(x^{-1}yx)^{-1} = C_A(xy^{-1}x^{-1}) = C_A(xyx^{-1}) = C_A(y)\ \Rightarrow x^{-1} \in N(A)$.

Proposition 3.20: A is a normal sub multigroup of B if and only if $N(A) = X$.

Proof: Let A be a normal sub multigroup of B, then $\forall\ n \in X$

$$C_A(n^{-1}xn) = C_A((n^{-1}x)n) = C_A((n^{-1}n)x) = C_A(e.x) = C_A(x)$$

Hence $C_A(n^{-1}xn) = C_A(x)$ and so $n \in N(A)$.

Therefore $N(A) = X$.

Conversely, suppose $N(A) = X, \forall\ x, n \in X$. We want to prove that A is a normal subgroup of B. Then $e \in A$, $C_A(e) = C_A(e.x)\ but\ e = n^{-1}n$

$$C_A((n^{-1}n)x) = C_A(n^{-1}xn)\ \forall\ x \in X, n \in N(A)$$

Hence $C_A(n^{-1}xn) = C_A(x)$.

That is A is a normal sub multigroup of B.

Remark: Let A be a normal sub multigroup of B, then $N(A) = M = N$.

If $M = \{x \in X /\ C_A(xy(yx)^{-1}) = C_A(e)\ \forall\ y \in X\}$ and

$N = \{x \in X /\ C_A(xy) = C_A(yx)\ \forall\ y \in X\}$.

Proposition 3.21: Let $M \in MG(X)$, and $M_1 \subseteq M$, $M_2 \subseteq M$, then $N(M_1) \cap N(M_1) \subseteq N(M_1 \cap M_2)$.

Proof: Let $y \in N(M_1)$ and $y \in N(M_2)$ which implies $y \in N(M_1)$ for any $y \in N(M_1) \cap N(M_1)$ for any $x.y \in X$, we get

$$C_{M_1 \cap M_2}(xyx^{-1}) = C_{M_1}(xyx^{-1}) \wedge C_{M_2}(xyx^{-1})$$

$$= C_{M_1}\big((xx^{-1})y\big) \wedge C_{M_2}((xx^{-1})y)$$

$$= C_{M_1}(e.y) \wedge C_{M_2}(e.y)$$

$$= C_{M_1}(y) \wedge C_{M_2}(y)$$

$$= C_{M_1 \cap M_2}(y)$$

Thus, $y \in N(M_1 \cap M_2)$.

Hence $N(M_1) \cap N(M_1) \subseteq N(M_1 \cap M_2)$.

Corollary 3.22: Let $M \in MG(X)$, and $M_1 \subseteq M, M_2 \subseteq M$, such that $C_{M_1}(e) = C_{M_2}(e)$. Then

$N(M_1) \cap N(M_1) = N(M_1 \cap M_2)$.

Proof: Recall that

$$N(M_1) = \{x \in X / C_A(xy) = C_A(yx) \; \forall \; y \in X\}$$
$$= \{x \in X / C_A(xyx^{-1}y^{-1}) = C_A(e) \; \forall \; y \in X\}$$

Let $y \in N(M_1 \cap M_2)$. Then by definition

$$C_{M_1 \cap M_2}(xyx^{-1}y^{-1}) = C_{M_1}(xyx^{-1}y^{-1}) \wedge C_{M_2}(xyx^{-1}y^{-1})$$
$$= C_{M_1}(e) \wedge C_{M_2}(e)$$
$$= C_{M_1 \cap M_2}(e)$$

Which implies $y \in N(M_2)$ and $y \in N(M_1)$. Thus $y \in N(M_1 \cap M_2)$ since

$C_{M_1}(xyx^{-1}y^{-1}) = C_{M_1}(e) \Rightarrow C_A(xy) = C_A(yx)$ and similarly in the case of $B, C_{M_1}(e) = C_{M_2}(e)$.

Hence $N(M_1) \cap N(M_1) = N(M_1 \cap M_2)$.

Corollary 3.23: Let $M \in MG(X)$, and $M_1 \subseteq M, M_2 \subseteq M$. Then

$N(M_1) \cap N(M_1) = N(M_1 o M_2)$.

Proof: Let $y \in N(M_1 \cap M_2)$, that is $y \in N(M_2)$ and $y \in N(M_1)$. Then for all $x \in X$

$$C_{M_1 o M_2}(y) = \vee\{C_{M_1}(a) \wedge C_{M_1}(b) \; \forall a, b \in X, y = ab\}$$
$$= \vee\{C_{M_1}(x^{-1}ax) \wedge C_{M_2}(x^{-1}bx) \; \forall a, b \in X, y = ab\}$$
$$= \vee\{C_{M_1}(c) \wedge C_{M_2}(d) \; \forall c, d \in X, x^{-1}yx = cd\}$$
$$= C_{M_1 o M_2}(x^{-1}yx)$$

$\Rightarrow C_{M_1 \cap M_2}(y) = C_{M_1 o M_2}(x^{-1}yx)$.

The inequality holds since$y = ab \Rightarrow x^{-1}abx = cd \Rightarrow ab = (xcx^{-1})(xdx^{-1})$ and since $a = xcx^{-1}$ and $b = xdx^{-1}$ implying $xax^{-1} = c$ and $xbx^{-1} = d$. Again

$$C_{M_1 o M_2}(x^{-1}yx) \leq C_{M_1 o M_2}(x(x^{-1}yx)x^{-1}) = C_{M_1 o M_2}(y)$$

So, $C_{M_1 o M_2}(y) \geq C_{M_1 o M_2}(x^{-1}yx)$.

Thus $C_{M_1 o M_2}(y) = C_{M_1 o M_2}(x^{-1}yx)$.

Hence $y \in\in N(M_1 o M_2)$.

Therefore, $N(M_1) \cap N(M_1) = N(M_1 o M_2)$.

Remark: Suppose $M \in MG(X)$, and $M_1 \subseteq M, M_2 \subseteq M$. If $M_1 \subseteq M_2$, then N($M_1) \subseteq N(M_2)$.

4. CONCLUSION

Centralizer and Normalizer of the classical group were studied under the perspective of multiset and an extended results from the existing ones were established. The closure of each of the concept under union, intersection, arithmetic addition, and arithmetic multiplication, composition were also studied. Finally, we have also shown that a centralizer and normalizer of a multi group is not empty and are sub multi group of a multi group. We have also define the normal sub multi group of these concepts and some results were put down. Other aspect of the classical group can also be studied under multiset perspecrtive.

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